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Decay Time of a Damped Pendulum

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Decay time of a Damped Pendulum

Introduction:

The goal for this lab was to devise an equation for the decay time of a damped pendulum released at varying angles. Decay time is not the time the pendulum takes to come to a complete stop, but rather when the swing gets to $\frac{1}{e}$(x) of its original magnitude. It was hypothesized that decay time would be greater with increasing angle. Initial velocity and initial angle will be proportional to each other. The higher the velocity of the pendulum, the less time the dampening mechanism will have to act against the pendulum motion. The initial angle of the pendulum will be carefully measured from the resting point of the pendulum. The equation:

$$v(t)=Ae^{-Bt}\sin(\omega t+C)+D \qquad \text{(eq. 1)}$$

stems from Newton's 2nd law: $\vec{F} = -k\vec{x}$, which can be rewritten in the form of the differential equation: $(x^{''}) + 2B\omega(x^{'}) + 2\omega^2(x) = \frac{F(t)}{m}$. A general solution for this equation can then be found, and rewritten as (eq. 1). This lab focuses on the B variable in this damped harmonic oscillator equation. The quantity τ (the value for $\frac{1}{B}$) is the damping constant, and essentially determines the behavior of the pendulum. As τ increases, the system will take longer to return to equilibrium. In this lab, we will be able to see if different factors affect τ and in what way. We will specifically focus on the initial angle of the pendulum.

Methods and Materials:

To obtain an answer for the hypothesis a careful analysis of the pendulum needs to be done using the computer program, Logger Pro 3.8.2. The set up should include a sturdy tower, rotary motion sensor model, CI:6625, a magnetic dampener with magnets on either side of the pendulum, and an aluminum spatula resembling solid pendulum.

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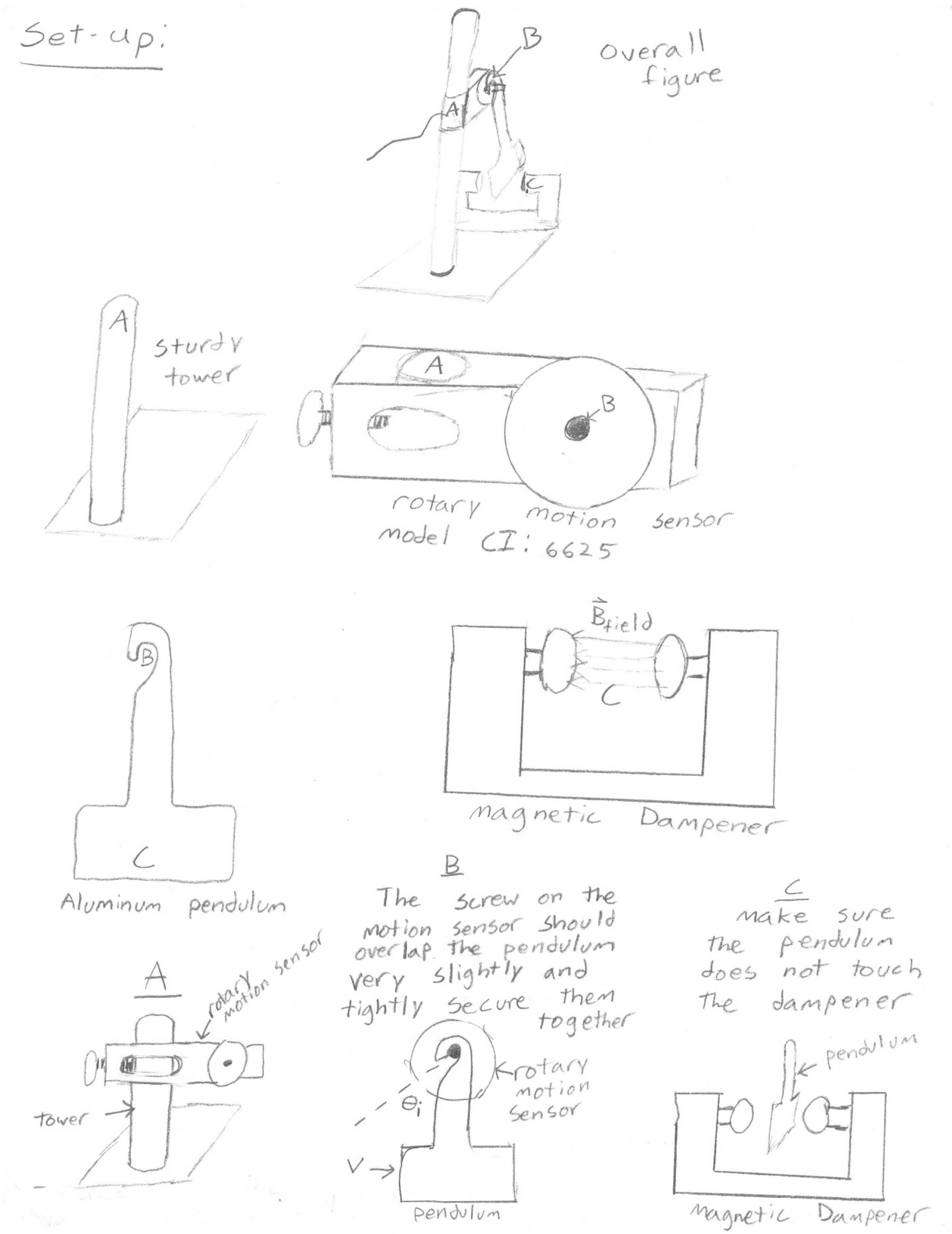

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This set up is to create a graph of the angle of the pendulum while it is swinging. Make sure to use a sturdy tower to minimize the number of outside variables. The rotary motion sensor will be mounted on the tower. The pendulum will be pinned on to the rotary motion sensor, the tighter this connection is, the better. The dampener has adjustable magnets that can be moved closer together for a stronger magnetic field or further apart for a weaker one. The pendulum should be set up so that the bottom rectangle swings through the middle of the magnets.

Type the equation v(t)=A*exp(-Bt)*sin(ω*t+C)+D as shown into Logger Pro v 3.8.2 to find the damping constant, tau, which in this case will be $\frac{1}{B}$.

A: amplitude

B: inverse of decay constant

ω: frequency

C: phase

D: y intercept

Results:

Table 1:

Angle (*)	**B (s)**	$\boldsymbol{\tau}$ $(\boldsymbol{s^{-1}})$
30	5.1 ± .4x10^-2	20 ± 1
60	4.6 ± .1x10^-2	22 ± 1
75	3.3 ± .1x10^-2	30 ± 1
90	4.9 ± .4x10^-2	20 ± 1
105	3.5 ± .3x10^-2	29 ± 1
120	4.1 ± .4x10^-2	25 ± 1
130	3.8 ± .3x10^-2	26 ± 1

This table compares the Decay Constant to a varying angle. The uncertainties were found using propagation of errors: $\frac{dT}{T} = \frac{dB}{B}$

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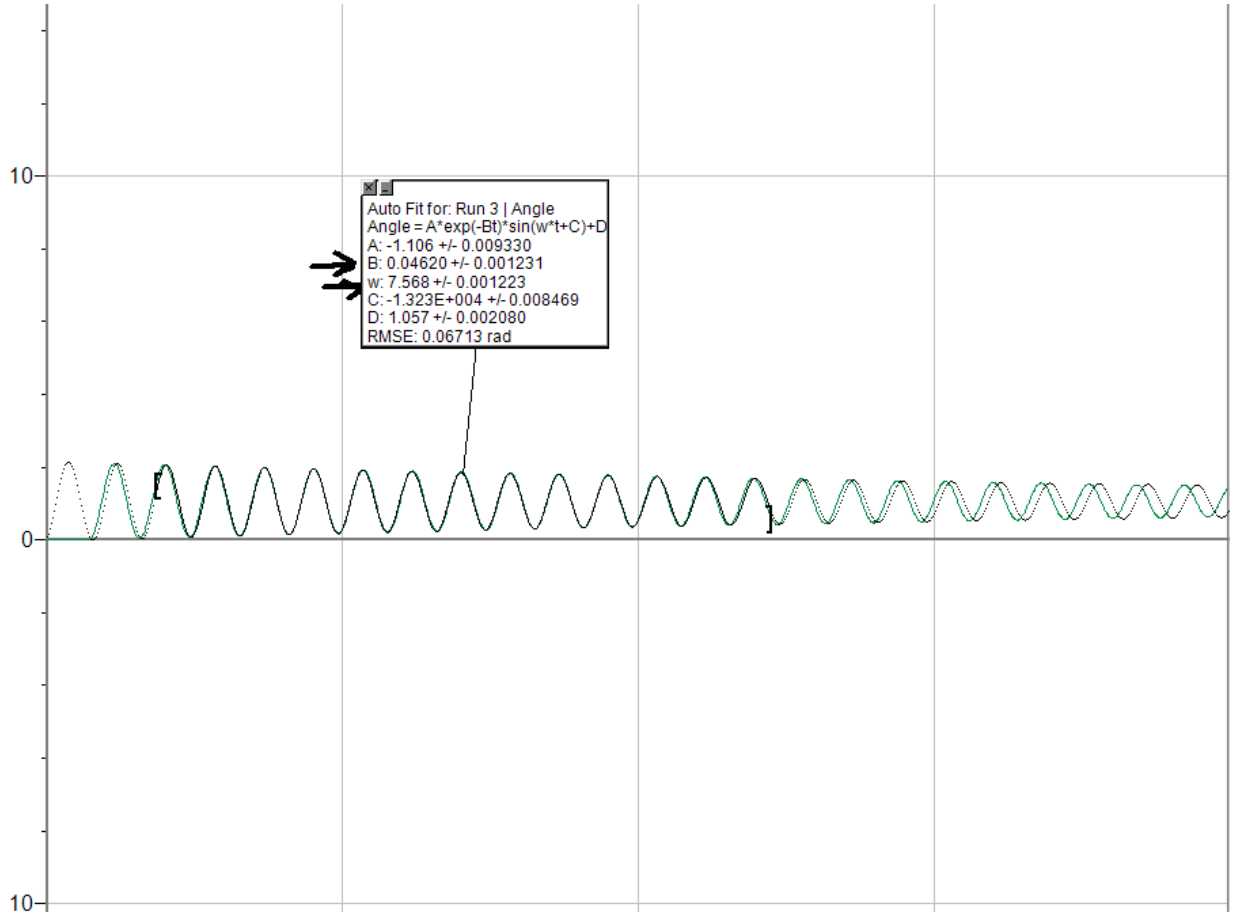
Graph 1:



This graph shows that the data in tables 1 and 2 were calculated using a fit line for the behavior of a pendulum.

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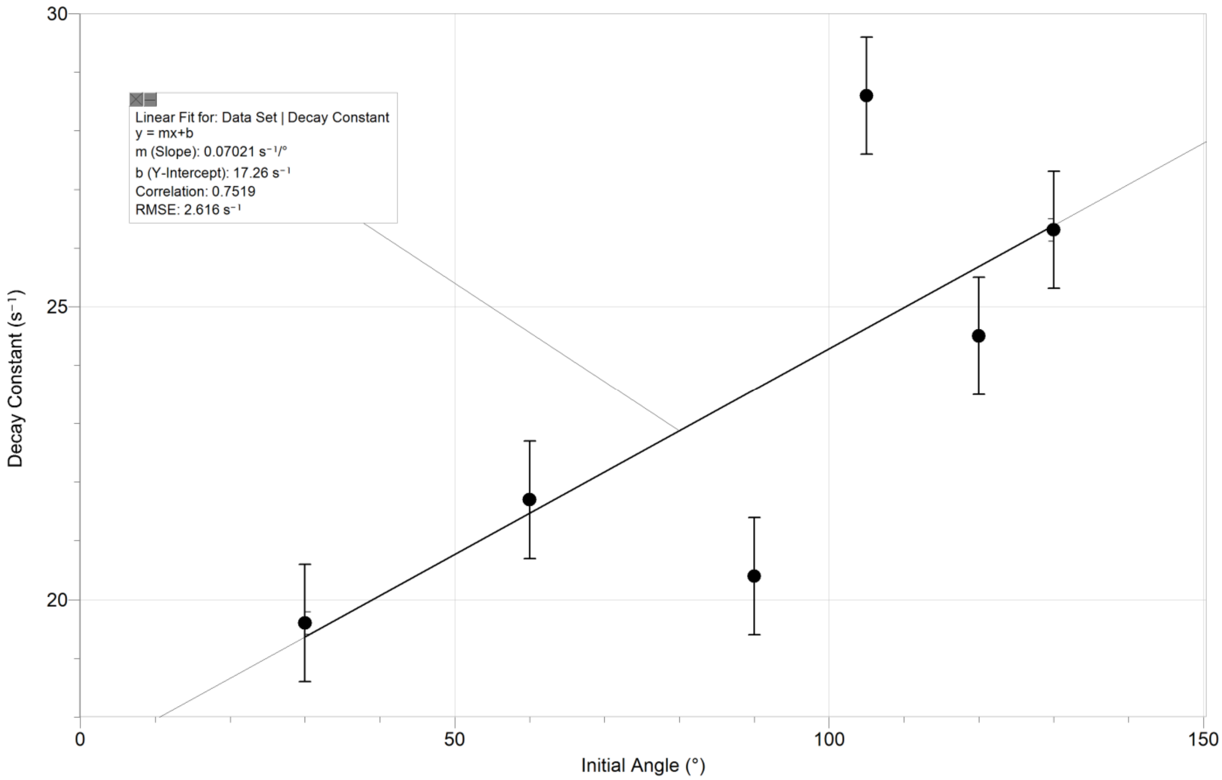
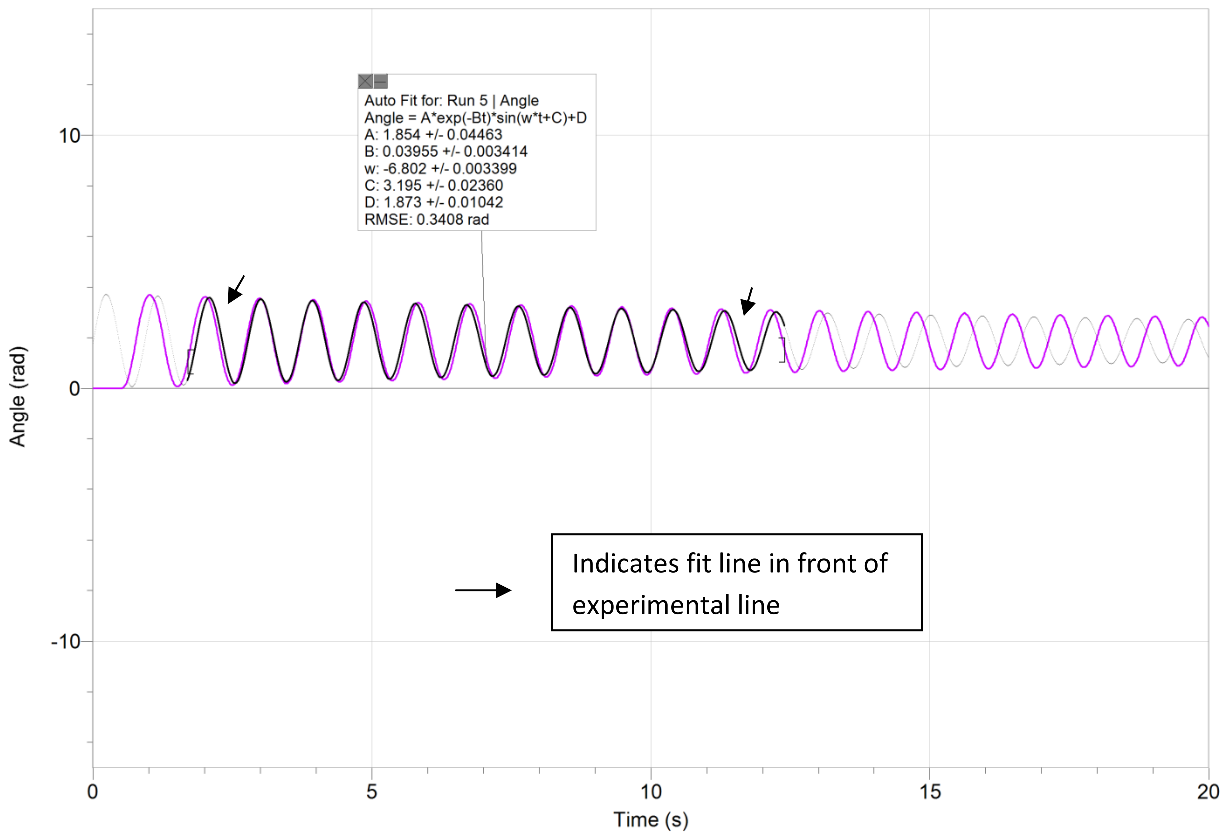
Graph 2:



This is a graph of the best linear fit of the decay constant as a function of the initial angle theta. The initial angle was calculated with respect to the resting position of the pendulum

Graph 3:



This is a graph of the velocity of the swinging pendulum, and will be referenced in the conclusion of this lab.

Table 2:

Angle (*)	ω (Hz)
30	7.809 ± .004
60	7.568 ± .001
75	7.366 ± .001
90	6.983 ± .003
105	6.802 ± .003
120	6.537 ± .004
130	6.246 ± .003

This table compares the frequency of the swing to the varying angle. The uncertainties were found using Logger Pro v 3.8.4.

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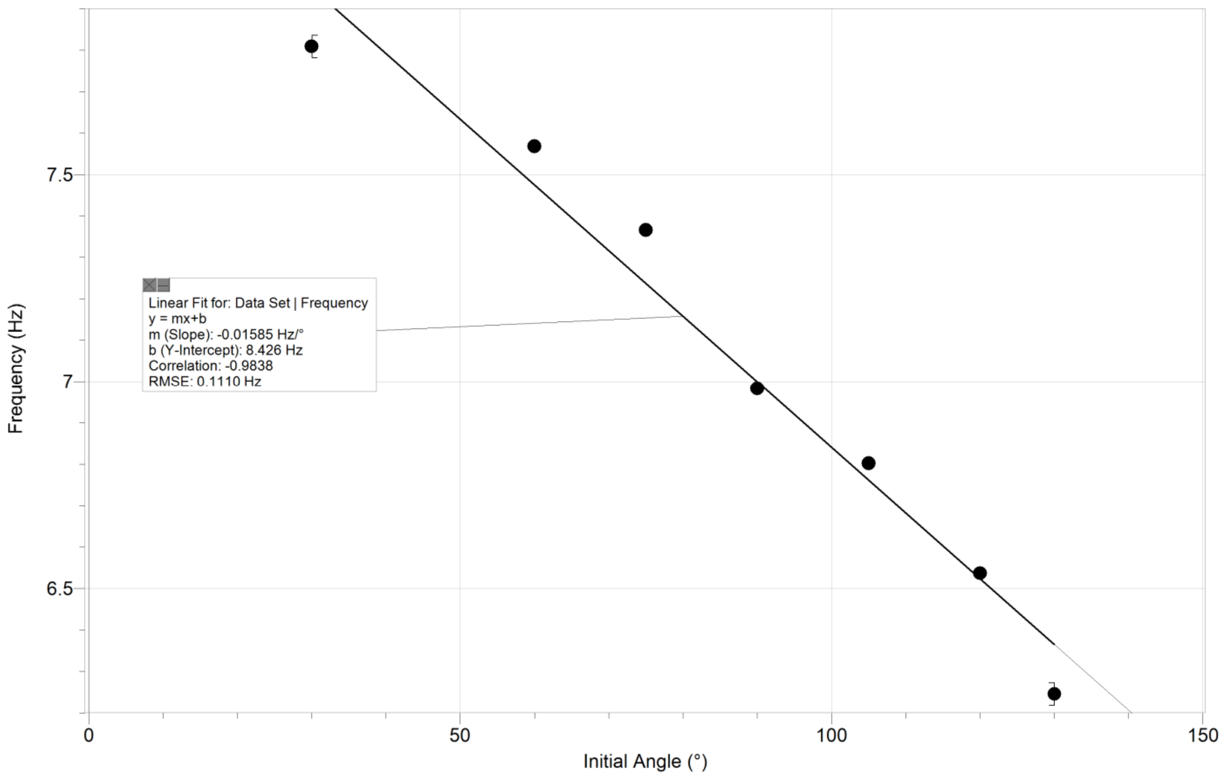
Graph 4:



This is a best linear fit graph of frequency as a function of theta. The calculated uncertainty is negligible.

Conclusion:

The goal for this lab was to devise an equation for the decay time of a damped pendulum. It was hypothesized that decay time would be greater with increasing angle. It was found that decay time does not depend on the varying angle (Graph 2). This may have been due to error in Logger Pro's fit line, or an unsteady tower. It also may have been due to a changing frequency within each run (Graph 3). This frequency was further analyzed to a minimal extent in this experiment. It was found that frequency is somewhat proportional to initial angle (Graph 4). I do not believe that either of these results are significant, but raise an interesting question about the changing frequency of a pendulum. The strange frequency shifts seemed to happen around 90* and above. A follow up experiment could be conducted investigating if this was due to some other error, or as a pendulum hits a certain velocity, if its individual frequency actually changes within its swinging motion.

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References:

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[2] Serway, Raymond A.; Jewett, John W. (2003). *Physics for Scientists and Engineers*. Brooks/Cole