UWB-FSK: Performance Tradeoffs for High and Low Complexity Receivers

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for High and Low Complexity Receivers

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Abstract — In this work we explore flexible modulations to allow demodulation using receivers with different complexity and cost. More specifically, we study pulse-based ultra wideband (UWB) communications over channels with dense multipath effects using frequency shift keying (FSK) data modulation. Our aim is to explore the performance tradeoffs between high and low complexity receivers. For this purpose we determine the performance of coherent, non-coherent and mismatched (e.g., non-coherent detection using templates consisting of a single path) demodulators. We take into account the influence of the frequency response of the antenna system and the effects of the frequency selectivity of the multipath channel. Given a specific channel condition, we derive expressions for the bit error rate for coherent, non-coherent and mismatched reception of correlated FSK signals with unequal energies, and calculate the signal-to-noise ratio (SNR) degradation for different receiver's complexities with a given FSK frequency deviation. We show that UWB-FSK has a SNR degradation similar to other low-complexity receivers studied in the literature.

Index Terms — UWB, FSK, multipath channels.

I. INTRODUCTION

Communications based on UWB for short-range high-speed wireless communications have been under intense study for several years [1]-[3], and has been proposed for HDTV distribution [4], home entertainment networks [5] with precise ranging [6], and digital multimedia networks [7].

An important aspect for consumer applications is to use flexible modulations to allow demodulation using receivers with different complexity and cost.

Different modulation schemes, e.g., pulse position modulation (PPM), pulse amplitude modulation (PAM), bi-phase modulation, etc., have been studied extensively (see for example [8]-[12]). Given that UWB has a large portion of the available spectrum for operation but it has restrictions in the amount of power that can be transmitted, it would make sense to consider modulations that exploit these properties.

It is well known (see for example chapter 9 in [14]) that orthogonal M-ary signals are power efficient, i.e., we can increase M improving the BER without increasing the signal power. Examples of orthogonal M-ary signals include PPM and FSK. The UWB systems, being power limited but not bandwidth limited (to a certain extent), can benefit from the use of orthogonal PPM or FSK signals. In fact, orthogonal PPM signals have been studied extensively [15],[16], and recent work has studied the use of frequency modulation (FM) for UWB [17]-[24].

In this work we study the performance of FSK UWB communications in the presence of AWGN and dense multipath effects (DME) [21]-[23]. One novelty in this work is to consider the combination of the antenna influence jointly with the frequency-selective multipath effects (this is not considered in previous works, see for example [17]-[20] and [25]). Our aim is to explore the performance tradeoffs between high and low complexity receivers, considering a more realistic scenario taking into account the combined effect of the influence of the frequency response of the antenna system and the effects of the frequency selectivity of the multipath channel.

The organization of this paper is as follows. In section II we discuss the relevant issues for FSK UWB. In Section III we introduce the FSK signals, and the models for the UWB pulse, antenna, and the multipath channel. In Section IV we describe the signal processing at the receiver and the BER expressions. Section V contains the numerical results.

II. UWB-FSK

A. Benefits of UWB-FSK

Similar to M-ary PPM, orthogonal M-ary FSK signals allow to increase M without reducing the bit transmission rate. Also, FSK allows to build M-ary communications signals that can provide a performance enhancement as M grows avoiding an M-fold increase in the complexity of the receiver. One possible drawback for PPM is that to increase M we need to increase the number of non-overlapping time slots. This can potentially reduce the bit rate. Besides, there is a practical limit on the maximum number of such slots that can be fitted in a frame time. Although for FSK the value of M would also be eventually limited, we could also consider to have the combination FSK-PPM [25] to get a combination of the benefits of both FSK and PPM.

The UWB-FSK has the advantage that the same transmitted signal can be demodulated with low complexity for low-price equipment, and with high complexity for high-price equipment. This is because non-coherent, and mismatched demodulation (i.e., one that does not require channel
estimation), can potentially reduce the receiver’s complexity at the expense of performance loss. Besides, there are more theoretical and practical benefits for using FSK in UWB, e.g., impulsive FSK UWB schemes with small duty cycle can achieve rates in the order of capacity [26], and FM-like UWB transmitted reference systems [27] don’t need a delay element, which is a difficult device to build.

B. Issues addressed for FSK-UWB

The channel’s frequency selectivity. Due to the frequency selectivity, the FSK signals corresponding to different channel realizations will arrive with different (random) energy. However, the mean energy averaged over many channel realizations will be similar for both FSK signals.

When comparing with previous results, we observe that in this work we are dealing with FSK signals propagating over a dispersive (sometimes characterized as log-normal) channel, which means that the FSK signals will arrive not only with a random any but also a random shape. The scenario in this work is different from the work done in [28], where the performance of non-coherent FSK in the presence of intersymbol interference (ISI) produced by dispersive Rayleigh channels is studied. More specifically, that work studied the effect of the r.m.s. delay spread on the ISI, developing error probability expressions for FSK pulses with 100% duty cycle. The 100% duty cycle results in signals with constant envelope, which facilitate the design of the analog front end. However, for low signaling rates it may require a large modulation index to satisfy the UWB requirements.

In the present work the UWB is obtained using a short duty cycle, not a large modulation index, which can resolve the multipath even for low signaling rates, and can potentially facilitate ranging calculations [29].

The antenna’s effect. It is known that the antenna system can modify the shape of the UWB received signal [30]. In the case of FSK, the antenna system can also modify the energy of the received signals. To illustrate this point, let us consider an antenna system that can be modeled as a derivative operation (this model was extensively studied in early work on UWB). For the signals considered in this work, there would be a difference in the received energy of

\[ 10 \log_{10} \left( \frac{f_c + \Delta}{f_c - \Delta} \right)^2 \]

(see section V), where \( f_c \) is the central frequency and \( \Delta \) is the frequency deviation of the FSK signals. Generalizing, we can see that if we use M-ary FSK, the maximum difference in energy would be

\[ 10 \log_{10} \left( \frac{f_c + \log_2(M)\Delta}{f_c - \log_2(M)\Delta} \right)^2 . \]

Although the system is linear, and the convolution of the antenna and channel is just another filter, the antenna effects cannot be simply included in the channel response. The reason is that the antenna is fixed, whereas the multipath is random and eventually is averaged.

The BER expressions. We consider signals with equal energy at the transmitter before the transmitting antenna, with the signals arriving with different (random) shapes, energies, and correlation, but with the same noise floor for all the signals. However, this complicates the derivation of BER expressions. In particular, the non-coherent BER expression are more complicated since the decoder is nonlinear, because of the envelope detection operation.

Previous BER results are derived for the equal-energy signal case (see for example [14] and [31]). In particular, the BER for two equal-energy and correlated Ricean random variables is determined in [31]. For the non-equal energy case, we had to derive the BER expressions using a methodology analogous to the one used in the equal-energy case. This derivation of the BER is elaborated, and due to the complexity of the expressions, in this work we focus on binary FSK. For certain scenarios (see section IV), the BER expressions derived in this work are exact.

Since the antenna response is known at the transmitter, using equal energy on the FSK signals at the transmitter might not make sense. After all, we could pre-compensate for the differences in energy at the transmitter and have the signals arrive with similar average energies. Actually, for coherent reception it would be equivalent either to receive signals with different energy and use an adaptive decision threshold, or to receive signals with the same energy and use an zero threshold.

However, even if the effect of the antenna is pre-compensated at the transmitter, the multipath effects will produce signals with different energy at the receiver (after the receiving antenna). In this work we investigate the effect of decoding signal with different (random) energies with a fixed threshold.

III. SYSTEM MODEL

Fig. 1 depicts the system model at the pulse level. In the following \( x(t) \rightarrow X(f) \) denotes a pair of Fourier transforms, where \( x(t) \) is an impulse response and \( X(f) \) is the frequency response, and PSD means power spectrum density.

Transmitted pulse, \( P_i^{TX}(t), \)

Antenna’s response \( h_i(t) \rightarrow H_i(f), \)

Kaiser Window \( h_R(t) \rightarrow W(f), \)

Channel response \( h_{CH}(\xi,t) \rightarrow H_{CH}(\xi,f), \)

Multipath-free pulse \( P_i(t) \rightarrow F_i(f), \)

Pulse with multipath \( P_i(t) \rightarrow F_i(\xi,t), \)

Matched filter \( h_i^M(\xi,t) \rightarrow H_i^M(\xi,f), \)

Mismatched filter \( h_i^{MM}(\xi,t) \rightarrow H_i^{MM}(\xi,f), \)

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for \( i = 1, 2 \), where \( n(t) \) is AWGN with a one-sided power spectrum density (PSD) \( N_o \). More details will be explained in the following sections.

\[
P_i^{TX}(t) = \sqrt{2E_a/T_0} e^{j2\pi(f_c+b\Delta)+\varphi}, \tag{1}
\]

for \( 0 \leq t \leq T_0 \), where \( E_a \) is the energy of the pulse, \( f_c = (q/T_o) \) is the center frequency, \( q \) is a positive integer, \( \Delta \) is a frequency shift, \( b \in \{-1, 1\} \) is the uniformly-distributed data bit, and \( \varphi \) is the phase of the oscillator used to produce the pulse, modeled as a random variable uniformly distributed over \([0, 2\pi]\). For \( P_i^{TX}(t) \) we use a complex-valued pulsed sine wave instead of the Gaussian monocycle because the pulsed sine wave is more appropriate for FSK and its bandwidth fits in both the FCC’s mask (3.1 to 10.6 GHz) [13] and the bandwidth of the multipath channel model (3.75 to 6.25 GHz) [33].

B. Effect of the antenna

There are several types of UWB antennas [34]-[35]. Under free propagation conditions, ignoring any nonlinear effects, and for the range of frequencies of interest, we model the effect of the antenna system using \( h_A(t)*h_A(f) \). Since the performance depends on the energy and correlation values of the received signals, this model is valid for any \( h_A(t) \).

C. Channel Model

We consider a slow time-varying residential indoor multipath channel stationary in the frequency domain. Since the receiver moves slowly, it can be assumed that the Doppler effect is negligible compared with the frequency shift \( \Delta \) used in the FSK signals.

We use an autoregressive (AR) model [36] to represent the small scale fading. The realizations of the random \( H_{CH}(\xi, f) \) is centered around 5 GHz, and corresponds to the output sequence of an infinite impulse response (IIR) filter (the \( \xi \) is a random variable indexing a certain realization \( H_{CH}(\xi, f) \) related to a certain propagation path between the fixed transmitter and the moving receiver.) The \( h_{CH}(\xi, t) \) is continuous, corresponding to DME. This channel model includes a random path loss (PL) model [37] to represent the large scale fading, with a Gaussian path loss exponent and lognormal shadowing. The PL model allows to evaluate bit error rate (BER) for different transmitter-receiver distance values \( D \). This channel model was first proposed in [36] [37] and later modified in [33]. The Kaiser window \( h_w(t) \) is centered at \( f_c \) and occupies a frequency band equal to the average bandwidth (3.75 to 6.25 GHz) of \( H_{CH}(\xi, f) \).

D. Received waveform

In the absence of multipath, the received pulse is

\[
P_i(t) = P_i^{TX}(t)*h_A(t)*h_w(t), \tag{2}
\]

for \( 0 \leq t \leq T_p \), \( (T_p > T_0 \) due to the filtering effect), where \( * \) denotes the convolution operation. In the presence of multipath, the received waveform

\[
\sqrt{E_a} P_i(\xi, t) = P_i(t)*h_{CH}(\xi, t), \quad \text{or} \quad \sqrt{E_a} F_i(\xi, f) = F_i(f) H_{CH}(\xi, f),
\]

with average duration \( T_a \) roughly equal to the mean delay spread of the channel, and with average received energy \( E_a \) (all the \( P_i(\xi, t) \) in the ensemble are normalized to have average energy equal to one).

E. Waveform Energy and Correlation values

The energy is

\[
E_i(\xi) = \frac{E_a}{2} \int_{-\infty}^{\infty} |F_i(\xi, f)|^2 df. \tag{3}
\]

Due to the antenna and channel effects, \( E_i(\xi) \neq E_2(\xi) \).

We now want to calculate the normalized correlation between \( F_i(\xi, f) \) (centered at \([f_c - \Delta]\)) and \( F_2(\xi, f) \) (centered at \([f_c + \Delta]\)) as a function of the frequency

\footnote{Although the model uses a Gaussian path loss exponent, precision computer simulations naturally avoids to have truly infinite values.}
separation \( \Delta_d = (+\Delta) - (-\Delta) = 2\Delta \). This complex-valued normalized correlation is given by\(^4\)

\[
Z_0(\xi, -\Delta, +\Delta) \Delta = \frac{E_d}{2} \times \int_{-\infty}^{\infty} \frac{F_1(\xi, f + \Delta_{\min}) F_2^*(\xi, f - \Delta_{\min}) df}{\sqrt{E_1(\xi, +\Delta_{\min}) E_2(\xi, -\Delta_{\min})}}
\]

\[
Z_0(\xi, -\Delta, +\Delta) = \sqrt{E_d} \{Z_0(\xi, -\Delta, +\Delta)\}, \quad \text{for } \Delta_{\min} = \Delta_{2nd}.
\]

where \( R_c \{x\} \) is the real part operator, and \( x^* \) denotes complex conjugate of \( x \). The \(|Z_0(\xi, -\Delta, +\Delta)|\) is the non coherent correlation, and the \( |Z_0(\xi, -\Delta, +\Delta)| \) is the coherent correlation. The \( Z_0(\xi, -\Delta, +\Delta) \) has its first zero crossing at \( \Delta_{1st} \) and its second zero crossing at \( \Delta_{2nd} \). The \(|Z_0(\xi, -\Delta, +\Delta)|\) has its first minimum at \( \Delta_{min,1} \approx \Delta_{2nd} \).

In free space the correlations are denoted \( Z_0(\Delta_d) \), and \( Z_r(\Delta_d) \). Due to the frequency selectivity of the channel, we are also interested in calculating the correlation values\(^5\)

\[
\begin{align*}
Z_{+\Delta_{\min}}(\xi, -\Delta_d) &= \frac{E_d}{2} \times \int_{-\infty}^{\infty} \frac{F_1(\xi, f + \Delta_{\min}) F_2^*(\xi, f + \Delta_{\min}) df}{\sqrt{E_1(\xi, +\Delta_{\min}) E_2(\xi, +\Delta_{\min})}} \\
Z_{-\Delta_{\min}}(\xi, -\Delta_d) &= \frac{E_d}{2} \times \int_{-\infty}^{\infty} \frac{F_1(\xi, f - \Delta_{\min}) F_2^*(\xi, f - \Delta_{\min}) df}{\sqrt{E_1(\xi, -\Delta_{\min}) E_2(\xi, -\Delta_{\min})}} \\
E_i(\xi, \pm \Delta_{\min}) &= \frac{E_d}{2} \int_{-\infty}^{\infty} |F_i(\xi, f + \pm \Delta_{\min})|^2 df.
\end{align*}
\]

These values help to write the BER expressions in a compact form.

Fig. 2 shows some examples of channel realizations using \( \Delta_{\text{mac}} = 0.3717 \) GHz. (a) Correlation functions, with \( \Delta_{\text{mac}} = 0.3717 \) GHz. (b) Waveforms in the frequency domain.

The bit rate is \( R_b = 1/T_b \). In multipath, the received signals are

\[
G_i(\xi, t) = \sum_{k=0}^{N_p-1} P_i(t - kT_f), \quad \text{for } i = 1, 2.
\]

These signals have duration \( T_f, N_pT_f \), with \( T_f \geq T_a \).

We define \( G_1(t) \) and \( G_2(t) \) as mismatched versions of the received signals \( G_1(\xi, t) \) and \( G_2(\xi, t) \), respectively, that are locally generated at the mismatched receiver.

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\textbf{G. Energy, correlation and cross-correlation}

The signals energies are

\[
E_{G_i}(\xi) = \frac{1}{2} \int_{-\infty}^{\infty} |G_i(\xi, t)|^2 dt \quad \text{(received)}
\]

\[
E_{G_i}(\xi) = \frac{1}{2} \int_{-\infty}^{\infty} |G_i(\xi, t)|^2 dt \quad \text{(mismatched)}
\]

for \( i = 1, 2 \). For notation convenience, we also define

\[
E_{G_i}(\xi) = \frac{E_{G_i}(\xi)}{E_{G_j}(\xi)}
\]

\( i, j = 1, 2 \),

\[
E_{G_i}(\xi) = \frac{E_{G_i}(\xi)}{E_{G_j}(\xi)}
\]

\( i, j = 1, 2, i \neq j \).

The normalized signal cross-correlations are

\[
R(\xi) = \frac{1}{2} \frac{\int_{-\infty}^{\infty} G_i(\xi, t) G_j(\xi, t) dt}{\sqrt{E_{G_i}(\xi) E_{G_j}(\xi)}},
\]

\[
R_\xi(\xi) = R_c \{R(\xi)\},
\]
\[
R = \frac{1}{2} \int_{-\infty}^{\infty} G_i(t) G_j^*(t) \frac{dt}{E_{G_i} E_{G_j}}, \tag{17}
\]
\[
\rho_{ij}(\xi) = \frac{1}{2} \int_{-\infty}^{\infty} G_{ij}(\xi,t) G_{ij}^*(t-\tau) \frac{dt}{E_{G_{ij}}(\xi) E_{G_{ij}}}, \tag{18}
\]
where \(\tau_i\) is a delay that maximizes
\[
\int_{-\infty}^{\infty} G_{ij}(\xi,t) G_{ij}^*(t-\tau) \frac{dt}{E_{G_{ij}}(\xi) E_{G_{ij}}} \tag{19}
\]

IV. RECEIVER SIGNAL PROCESSING AND PERFORMANCE

A. Signal Processing
The complex-valued received signal is
\[
z(\xi,t) = G_i(\xi,t) + n(t), \quad i = 1,2. \tag{20}
\]
Signal detection is achieved using perfectly-matched filters for coherent detection, perfectly-matched filters followed by envelope detectors for non-coherent detection, and mismatched filters followed by envelope detectors for mismatched detection (see fig. 1). We assume a receiver perfectly synchronized in time, frequency and phase, but notice that the phase information is lost in the envelope detection process for non-coherent detection.

The demodulation problem can be analyzed as the detection of 2 correlated, unequal-energy, equally-likely signals in the presence of AWGN. We start from well known results for equal-energy correlated signals \([14,31]\), and generalize these results to our unequal-energy case (in the non-coherent and mismatched cases the generalization is non-trivial). We first calculate the BER conditioned on a given channel condition \(\xi\), and then average over the channel effects. Due to limited space, only final expressions are given.

B. Performance for coherent detection
For coherent detection a perfectly synchronized Rake Receiver will have 2 filters matched to \(G_i(\xi,t), i = 1,2\), or alternatively, one filter matched to \(G_2(\xi,t) - G_1(\xi,t)\), and the decision variable is given by
\[
y_i(\xi) = \int_{0}^{h} z(\xi,t)[G_2(\xi,t) - G_1(\xi,t)]^t \frac{dt}{E_{G_2}(\xi) - E_{G_1}(\xi)}. \tag{21}
\]
For equally likely signals the BER is \(\frac{1}{2} P_e(0|1) + \frac{1}{2} P_e(1|0)\), where \(P_e(0|1)\) is the probability that \(y_1^2 - y_2^2 < 0\) given that a 1 was transmitted, and \(P_e(1|0)\) is the probability that \(y_1^2 - y_2^2 < 0\) given that a 0 was transmitted. We assume a fixed decision threshold \(T = 0\), and \(E_{G_2}(\xi) > E_{G_1}(\xi)\). The conditioned BER is
\[
Q(a_1, b_1) - \frac{c_1 - I_0(a_1 b_1) e^{-\frac{1}{2}(a_1^2 + b_1^2)}}{1 + c_1} + \frac{2}{2}, \tag{26}
\]
where the parameters \(a_i, b_i, c_i, i = 1,2\), all depend on \(\xi\), and are given in the appendix.

C. Performance for mismatched detection
Signal detection is achieved by cross correlating \(z(\xi,t)\) with \(G_i(t), j = 1,2\), followed by envelope detectors for non-coherent detection. The outputs of the envelope detectors are
\[
y_i(\xi) = \int_{0}^{h} z(\xi,t) G_i^*(t-\tau) \frac{dt}{E_{G_i}(\xi) E_{G_i}} \tag{25}
\]
The mismatched signals \(G_i(t)\) and \(G_2(t)\) are locally generated without channel estimation, and the only parameters that need to be estimated are the delays \(\tau_i\) and \(\tau_2\) in (25).

For equally likely signals BER is \(\frac{1}{2} P_e(0|1) + \frac{1}{2} P_e(1|0)\), where \(P_e(0|1)\) is the probability that \(y_2^2 - y_1^2 < 0\) given that a 1 was transmitted, and \(P_e(1|0)\) is the probability that \(y_1^2 - y_2^2 < 0\) given that a 0 was transmitted. We assume a fixed decision threshold \(T = 0\), and \(E_{G_2}(\xi) > E_{G_1}(\xi)\). The conditioned BER is
\[
Q(a_2, b_2) - \frac{c_2 - I_0(a_2 b_2) e^{-\frac{1}{2}(a_2^2 + b_2^2)}}{1 + c_2} + \frac{2}{2}, \tag{26}
\]
where the parameters \(a_i, b_i, c_i, i = 1,2\), all depend on \(\xi\), and are given in the appendix.

D. Performance for non-coherent detection
For non coherent detection the receiver will have 2 filters matched to \(G_i(\xi,t), i = 1,2\), followed each by an envelope detector. The outputs of the envelope detectors are given by
\[
y_i(\xi) = \int_{0}^{h} z(\xi,t) G_i^*(t) \frac{dt}{E_{G_i}(\xi) E_{G_i}} \tag{27}
\]
energies, with \(E_{G_2}(\xi) > E_{G_1}(\xi)\). The conditioned BER can be expressed as
\[
BER(\xi) = \frac{1}{2} Q \left( \frac{\epsilon_{G_2}(\xi) - T(\xi)}{\sigma_{\epsilon_2}(\xi)} \right) + \frac{1}{2} Q \left( \frac{T(\xi) - m_{G_1}(\xi)}{\sigma_{m_1}(\xi)} \right), \tag{22}
\]
where \(Q(\cdot)\) is the Gaussian-tail integral, and where
\[
\begin{align*}
T(\xi) - m_{G_1}(\xi) & = \frac{\epsilon_{G_2}(\xi)}{N_u} - \frac{E_{G_2} - \epsilon_{G_1}(\xi)}{E_{G_2} - \epsilon_{G_1}(\xi)} \left[ E_{G_2} - \epsilon_{G_1}(\xi) \right]. \tag{23} \\
\epsilon_{G_2}(\xi) - T(\xi) & = \frac{\epsilon_{G_2}(\xi)}{N_u} - \frac{E_{G_2} - \epsilon_{G_1}(\xi)}{E_{G_2} - \epsilon_{G_1}(\xi)} \left[ E_{G_2} - \epsilon_{G_1}(\xi) \right]. \tag{24}
\end{align*}
\]
For equally likely signals $BER=\frac{1}{2}P_e(0|1) + \frac{1}{2}P_e(1|0)$, where $P_e(0|1)$ is the probability that $y_2^2 - y_1^2 < T$ given that a 1 was transmitted, and $P_e(1|0)$ is the probability that $y_1^2 - y_2^2 < T$ given that a 0 was transmitted. We have assumed a fixed decision threshold $T = 0$, and $E_{G_2}(\xi) > E_{G_1}(\xi)$. The conditioned BER for non-coherent detection can be obtained as a special case of the mismatched case. More specifically, for perfectly matched receiver filters we have that $E_{G_1} = E_{G_2} = 1$, $\rho_{12} = R^*$, $\rho_{21} = R$, and $\rho_{11} = \rho_{22} = 1$. The resulting expressions for the parameters $a_i, b_i, c_i, i = 1,2$, all depending on $\xi$, are given in the appendix.

E. Validity of the BER expressions

In this section we discuss some issues related to the validity of the BER expressions.

**Interference Issue.** We consider signals with $T_f > T_p > T_0$. Hence, we assume that inter-pulse interference (IPI) can be considered negligible. When IPI is small and/or $N_p$ is large, then inter-symbol interference (ISI) can also be considered negligible. If the mismatched signals $G_i(t)$ consist of a single path, then the self interference (SI) [31] can be considered negligible. If $G_i(t)$ consists of several paths, then the SI caused by the non-resolvable paths not being captured by the receiver must be taken into account.

If all IPI, ISI, and SI can be neglected, then BER expressions would be exact. If IPI or ISI do exist, the BER expressions should be treated as matched filter bound results for the coherent and non-coherent case. If SI do exist, the BER expressions should be treated as a lower bound for the mismatched case. Depending on the SI being Gaussian-like or not, this lower bound can be overly optimistic.

**Simplification for the coherent case.** For the case of signals with equal energy $E_{G_2}(\xi) = E_{G_1}(\xi) = E_G(\xi)$, we have $E_{21} = E_{12} = 1$, and it is straightforward to verify that in this case the expression in (22) simplifies to the standard result in [9], namely

$$BER(\xi) = Q\left(\frac{m_2(\xi)}{\sigma(\xi)}\right),$$

where $\left(\frac{m_2(\xi)}{\sigma(\xi)}\right) = \sqrt{E_G(\xi)/N_0}\sqrt{\frac{1}{1-R_c(\xi)}}$.

**Simplification for the non-coherent case.** As mentioned before, the conditioned BER for non-coherent detection can be obtained from the mismatched case by making the values $E_{G_1} = E_{G_2} = 1$, $\rho_{12} = R^*$, $\rho_{21} = R$, and $\rho_{11} = \rho_{22} = 1$, resulting in the expressions given in the Appendix.

Furthermore, for the case of signals with equal energy

$$E_{G_2}(\xi) = E_{G_1}(\xi) = E_G(\xi)$$

we have that $E_{21} = E_{12} = 1$, and it is straightforward to verify that in this case the BER expression (26) simplifies to the standard result in [31], namely

$$BER(\xi) = Q(a,b) - \frac{1}{2}I_0(ab)e^{\frac{1}{2}(a^2+b^2)}$$

where

$$a = \sqrt{\frac{E_G(\xi)}{2N_o}}\left[1 - \sqrt{1 - R^2}\right], b = \sqrt{\frac{E_G(\xi)}{2N_o}}\sqrt{1 + \sqrt{1 - R^2}}.$$

**F. Average over multipath.**

The averaged performance can be obtained by taking the expected value $E_x\{\}$ of (22) or (26) over all values of $\xi$ to get $BER(E_x/N_o) = E_x\{BER(\xi)\}$, where $E_b$ is the average bit energy.

V. NUMERICAL RESULTS

For $P_i^{TX}(t)$ in (1) we use $q = 15$ and $T_o = 3.0$ ns to get a central frequency $f_c = 5$ GHz. For the phase we set $\Phi = 0$ radians. For $P_i(t)$ (free-space propagation case) we get $T_p \cong 4$ ns with a spectrum $F_\tau(f)$ centered at $f_c \pm \Delta$ GHz, with a 10 dB bandwidth (BW) of about 480 MHz.

Although each individual UWB pulse does not strictly satisfy the FCC's definition requiring BW $\geq 500$ MHz, the signals after FSK modulation do (see fig. 2b). The coherent $Z_r(\Delta_d)$ has the first zero-crossings at $\Delta_{1r} \cong 0.1667$ GHz, it has the second zero-crossings at $\Delta_{2nd} \cong 0.3717$ GHz, and has a minimum value $Z_r(0.25 \cdot 10^9) \cong -0.2942$. The non-coherent $|Z_0(\Delta_d)|$ has the first minimum at $\Delta_{min} \cong 0.3717$ GHz. We select the frequency shift value $\Delta$ related to the first minimum of $|Z_0(\Delta_d)|$, namely

$$\Delta = \Delta_{min}/2 \cong 0.1858 \text{ GHz},$$

(30)

to get signal correlation values equal to zero, but notice that in multipath the correlation values are not necessarily zero.

For illustration purposes, the influence of the antenna system is modeled with a derivative operation (this antenna system model was repeatedly used in early works, e.g., [1] [15] [16]). Notice that with this antenna model, $P_i(t)$ and
$P_1(t)$ will have the same shape, but they are received with different energies, this difference in energy is 

$$10 \log_{10} \left( \frac{f_e + \Delta}{f_e - \Delta} \right) \approx 0.6458 \text{ dB}$$ \hspace{1cm} (31)

The BER expressions are still valid for other types of antennas [30] that reflect the pulse distortion during propagation [32], because these expressions are based on the energy and correlation values of the received signals.

For the multipath case we consider both line-of-sight (LOS) and not-line-of-sight (NLOS) scenarios. The set of received UWB waveforms $P_i(t)$ is generated using the autoregressive channel model in [33] to form and ensemble of channel pulse responses, with a sample size of 200 for every distance $D$, and averaging the results over $D = 3.9$ meters for LOS and $D = 1.3$ meters for NLOS. These UWB waveforms have an average delay spread $T_d \approx 160$ ns. Fig. 2b depicts some examples of channel realizations showing the effects of the channel frequency selectivity.

For the mismatched signals $G_i(t)$ we use the single-path signals equal to the received signals in free space, but other forms of mismatched signals with multiple paths are possible. The delays $\tau_1$ and $\tau_2$ in (25) are calculated using waveforms with no noise.

The BER expressions (22) for coherent, and (26) for non-coherent, are verified in AWGN by performing a Monte Carlo simulation, showing a good fit between simulated and theoretical results (as it should be, since the simulation conditions have neither IPI nor ISI or SI, hence the BER expressions are exact).

Evaluation of BER in (22) and (26) in multipath is done using a quasi-analytical method as in [38] in which the communications waveforms are not modeled analytically but numerically generated using the channel model in [33]. For every channelization we calculate the energy and correlation values in (11)-(18), and use these values in either (22) or (26) to find a function $BER(\xi)$. The expected values $E_\xi \{ BER(\xi) \}$ is then approximated with a sample average over the different realizations of the channel.

Fig. 3 shows the BER plots for coherent FSK with adaptive threshold $T$, coherent FSK with fixed $T = 0$, non-coherent FSK with fixed $T = 0$, and non-coherent mismatched FSK with fixed $T = 0$.

**VI. DISCUSSION AND CONCLUSIONS**

This work studied UWB communications over dense multipath channels using binary FSK data modulation. An interesting advantage of FSK is that the same transmitted FSK can be decoded using either coherent or non-coherent receivers, allowing a trade-off between performance and complexity.

In our analysis we take into account the influence of the frequency response of the antenna system and the effects of the frequency selectivity of the multipath channel.

![Fig. 3. BER vs. $\left( E_b / N_0 \right)$ for coherent (C), non-coherent (NC) and mismatched (MIS) using analytic evaluation, Monte Carlo simulation (simul), and Quasi-analytic evaluation. (a) BER in AWGN by analytic and MC simulation. (b) BER in AWGN by QA. (c) BER in LOS by QA. (d) BER in NLOS by QA. The BER for PAM and PPM results [28] are included for reference.](image)

In the absence of inter-pulse interference, inter-symbol interference, and self-interference, the BER expressions can be considered exact, and they simplify to standard results when signals with equal energy are considered. Due to the antenna effects, both FSK signals are received with unequal average energy. With the antenna modeled by a derivative operation, and under ideal propagation, this difference in energy is 0.6458 dB for an FSK frequency deviation $\Delta \approx 0.1858$ GHz. Due to the frequency selectivity of the multipath channel, both FSK signals are received with different (random) energies.

Table 1 compares the approximate SNR degradation for different non-coherent receivers with respect to their corresponding coherent receivers.

**TABLE I**

**APPROXIMATE SNR DEGRADATION FOR DIFFERENT NON-COHERENT UWB RECEIVERS: DELAY-BASED (CONVENTIONAL) TRANSMITTED-REFERENCE (TR), SLIGHTLY FREQUENCY-SHIFTED (TRANSMITTED) REFERENCE (SFSR), DIFFERENTIAL RECEIVER (DR), AND MISMATCHED FSK (MIS-FSK), (MIS-FSK REFERS TO THIS WORK).**

<table>
<thead>
<tr>
<th>Type of receiver</th>
<th>SNR degradation</th>
<th>Channel type</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFSR [27]</td>
<td>7.0 dB</td>
<td>Gaussian exponential decay channel</td>
</tr>
<tr>
<td>TR [40]</td>
<td>7.5 dB</td>
<td>Modified IEEE 802.15.3 channel</td>
</tr>
<tr>
<td>DR [40]</td>
<td>7.5 dB</td>
<td>Modified IEEE 802.15.3 channel</td>
</tr>
<tr>
<td>mis-FSK (LOS)</td>
<td>8.7 dB</td>
<td>Modified AR LOS channel in [33]</td>
</tr>
<tr>
<td>TR [39]</td>
<td>9.0 dB</td>
<td>Rayleigh channel</td>
</tr>
<tr>
<td>mis-FSK (NLOS)</td>
<td>10.7 dB</td>
<td>Modified AR NLOS channel in [33]</td>
</tr>
</tbody>
</table>
Further studies need to be done considering M-ary FSK with large $M$ and mismatched FSK UWB using templates composed of several paths.

**APPENDIX**

The parameters $a_i$, $b_i$, $c_i$, $i = 1, 2$, for the BER in (26) (mismatched case) are given in the following expressions:

\[
a_i = \frac{E_{g_1}(\xi)}{N_o} \cdot \frac{1}{2A_i} \times \frac{\rho_{11}^2 + \rho_{21}^2 - 2 R_i \rho_{12} \rho_{21} R_i}{(1 - |R_i|^2)} \frac{1}{2} \frac{(E_{g_1} |\rho_{11}^2 - E_{g_2} |\rho_{21}^2 R_i)}{(1 - |R_i|^2)} B_i
\]

\[
b_i = \frac{E_{g_1}(\xi)}{N_o} \cdot \frac{1}{2A_i} \times \frac{\rho_{11}^2 + \rho_{21}^2 - 2 R_i \rho_{12} \rho_{21} R_i}{(1 - |R_i|^2)} \frac{1}{2} \frac{(E_{g_1} |\rho_{11}^2 - E_{g_2} |\rho_{21}^2 R_i)}{(1 - |R_i|^2)} C_i
\]

\[
c_i = \frac{(E_{g_1} + E_{g_2})^2 + 4E_{g_1}E_{g_2}(1 - |R_i|^2)}{(E_{g_1}E_{g_2})^2} + (E_{g_1} - E_{g_2}),
\]

and

\[
a_2 = \frac{E_{g_2}(\xi)}{N_o} \cdot \frac{1}{2A_2} \times \frac{\rho_{22}^2 + \rho_{12}^2 - 2 R_i \rho_{21} \rho_{12} R_i}{(1 - |R_i|^2)} \frac{1}{2} \frac{(E_{g_2} |\rho_{22}^2 - E_{g_1} |\rho_{12}^2 R_i)}{(1 - |R_i|^2)} B_2
\]

\[
b_2 = \frac{E_{g_2}(\xi)}{N_o} \cdot \frac{1}{2A_2} \times \frac{\rho_{22}^2 + \rho_{12}^2 - 2 R_i \rho_{21} \rho_{12} R_i}{(1 - |R_i|^2)} \frac{1}{2} \frac{(E_{g_2} |\rho_{22}^2 - E_{g_1} |\rho_{12}^2 R_i)}{(1 - |R_i|^2)} C_2
\]

\[
c_2 = \frac{(E_{g_2} - E_{g_1})^2 + 4E_{g_2}E_{g_1}(1 - |R_i|^2)}{(E_{g_1}E_{g_2})^2} + (E_{g_2} - E_{g_1}).
\]

Similarly, the parameters $a_i$, $b_i$, $c_i$, $i = 1, 2$, for the BER in the non-coherent case are given in the following expressions:

\[
a_i = \frac{E_{g_1}(\xi)}{N_o} \cdot \frac{1}{2A_i} \times \frac{1}{2} \frac{(1 - |E_{g_1}| |R(\xi)|^2)}{1 - |R(\xi)|^2} B_i
\]

\[
b_i = \frac{E_{g_1}(\xi)}{N_o} \cdot \frac{1}{2A_i} \times \frac{1}{2} \frac{(1 - |E_{g_1}| |R(\xi)|^2)}{1 - |R(\xi)|^2} C_i
\]

\[
c_i = \frac{(1 - E_{g_1})^2 + 4E_{g_1}(1 - |R(\xi)|^2)}{(E_{g_1})^2} + (1 - E_{g_1}),
\]

and

\[
a_2 = \frac{E_{g_2}(\xi)}{N_o} \cdot \frac{1}{2A_2} \times \frac{1}{2} \frac{(1 - |E_{g_2}| |R(\xi)|^2)}{1 - |R(\xi)|^2} B_2
\]

\[
b_2 = \frac{E_{g_2}(\xi)}{N_o} \cdot \frac{1}{2A_2} \times \frac{1}{2} \frac{(1 - |E_{g_2}| |R(\xi)|^2)}{1 - |R(\xi)|^2} C_2
\]

\[
c_2 = \frac{(1 - E_{g_2})^2 + 4E_{g_2}(1 - |R(\xi)|^2)}{(E_{g_2})^2} + (1 - E_{g_2}).
\]

**REFERENCES**


