Mathematical Practice and Human Cognition

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Mathematical Practice and Human Cognition
Remarks on Quinn’s “Science of Mathematics”

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Overview

- Introduction: Remark on mathematical practice
- Frank Quinn’s Contributions (to a Science of Contemporary Mathematics)

Mathematical Concepts
- A historical example
- Mathematical practice: Defining concept (FQ)
- Mathematical practice: Acquisition of concepts (FQ)
- Mathematical practice: Corroboration by math ed (cogn. sci.)
- *Mathematical practice: convergence with phenomenology
Introduction: Remark on mathematical practice (MP)

Three meanings of MP

- MP in Math Ed and PME


  - Deliberate inclusion of insights from various disciplines
Introduction: Remark on mathematical practice (MP)

MP as a culture

▶ “that complex whole that includes knowledge, belief, art, morals, law, custom and any other capabilities and habits acquired by [mathematicians] as members of [their trade].”
Edward Tylor, *Primitive Culture* (1871), vol. 1, p. 1
Frank Quinn: Relevant Publications

- 1992: “Theoretical Mathematics” (BAMS; jointly w/ A. Jaffe)
- 2012: “Revolutions in Mathematics?” (NAMS)

- 2011: Contributions to a Science of Mathematics

Quinn’s three periods

- I. ??–1600: “qualitative and philosophical”
- II. 1600–late 19th c: “quantitative and mathematical”
  scientific needs; elite practioner syndrome
- III. late 19th c through Hilbert’s Göttingen–??
  ontologically autonomous, methodologically unique
Quinn’s Third Period: Rigor

- **Traumatic transition**
  “the changes were forced by [the] increasingly difficulty of the mathematics and [the] ambition of the profession.”

- **Methodology**
  Rigorous definitions along with “genuinely error-displaying methods” secure the “complete reliability” of all mathematical conclusions.

- “The slavish devotion of mathematicians to rigorous methodology is required by the subject […] Rigor plays the same role in mathematics that agreement with the physical world plays in other sciences. Relaxing rigor is like ignoring data.”
Concepts: Continuity as an example

- Period I. Philosophy and application: Leibniz’ principle of continuity

- Period II. Quantitative and mathematical: $\epsilon$-$\delta$ approach
  Cauchy, building on d’Alembert, Euler, Lagrange, followed by
  Bolzano, Dedekind, Weierstrass

- Period III. Purely mathematical: topological definition
  Maurice Fréchet, Frigyes Reisz (not Marcel), Felix Hausdorff,
  Kazimierz Kuratowksi, among others
Concepts: Continuity a a topological notion

**Definition.** A topological space $\langle X, T \rangle$ is a set $X$ together with a topology $T$, i.e., a family of open subsets of $X$, such that

1. $\emptyset$ and $X$ are both open,
2. arbitrary unions of open sets are open,
3. finite intersection of open sets are open.

**Definition.** A function $f : S \to T$ between two topological spaces is continuous iff the pre-image $f^{-1}(Q)$ of every open set $Q \subset T$ is an open set $P \subset S$. 
Concepts: Quinn’s question

- Increase in rigor and loss of experiential or intuitive contents result in a concentration on the mathematical substance.

- Definitions are *not* simply a codification of an intuitive understanding” but “were developed and refined over long periods and with great effort,” and were, in fact, “frequently a community effort.”

- Quinn’s question: How do human agents acquire such concepts?
Concepts: Quinn’s answer

1. Sever as many ties to ordinary language as possible and limit ordinary language explanations to an absolute minimum

2. Introduce axiomatic definitions and bundle them up with a sufficient number of examples, lemmata, propositions, etc. into small cognitive packages

3. Have students practice hard with one new cognitive package at a time

4. Lather, rinse, repeat.
Concepts: How did it evolve and into what?

- Natural selection: those who did adopt another approach could no longer compete and eventually sank into oblivion

- Outcomes:
  - Core mathematics (vs mathematical sciences)
  - Empowering rank-and-file faculty (vs elite-practioner)
  - Mathematical altruism: faculty develop habits that support and nurse such practices of conceptual and methodic rigor
Quinn: Is he right?

- Soft empirical evidence
  - Quinn’s own expert testimony
  - Graduate level textbooks

- Hard empirical evidence?
  - Well, 2nd part of the talk ;-)
Digression: PoMP & Quinn is right?

- Traditional PoM reduced cognitive labor to deductive proof
  - Legacy of logicism

- A PoMP may realize that such a reduction is wrong
  - Philosophy becomes richer and much more complex
Evidence from CS, MathEd, PME: Caution

- Caution re neuroimaging: It’s too early to tell

- Caution re MathEd/PME
  - Undue influence of P&P
  - No focus on advanced mathematics
  - Lack of empirical reliability: sample sizes, reproducibility
Supporting evidence from MathEd/PME (and CS)

Quinn’s No 1: Sever ties to ordinary language

- From lexical decision task to priming: fact or fiction? (eg, Kahneman 2012 letter)
- Embodied knowledge and met-befores (eg, Tall 2008, 2013)
- Generic extension principle & epistemic obstacles (eg, Tall 1986; Cornu 1982, Sierpińska 1985ab)
Supporting evidence from MathEd/PME (and CS)

Quinn’s No 2: Cognitive packaging: definitions plus exercises

- Adding properties (ie, meaning) and fluidity (ie, mastery) (eg, Dreyfus 1991)
- Concept definition vs concept image (eg, Vinner 1983, 1991)
- CS: Package size matters (eg, Anderson&Lee&Fincham 2014); inhibition control (eg, Houdé et al., op cit.)
Supporting evidence from MathEd/PME (and CS)

Quinn’s No 3: Practice hard!

1. Automation: load issues (eg, Thurston 1990: compressibility; Lee&Ng&Ng 2009: word problems)

2. Mathematical “Habits of Mind”
   (eg, Selden&Lim 2010; Wilkerson-Jerde&Wilensky 2009, 2011: novices vs experts)
Qualifying evidence

- Contradicting evidence
  - Different cultures: Mathematicians responding to Jaffe-Quinn
  - Tall 2013: Introduction

- Enriching evidence
  - Studies that lend support for Quinn’s thesis also provide a much richer, higher-resolution picture of the cognitive processes involved
Enriching perspectives from MathEd/PME (and CS)

Quinn’s No 1: Sever ties to ordinary language

- Continuity and motivation:
  conceptual-embodied – proceptual-symbolic – axiomatic formal (eg, Tall 2008, 2013)

- Continuity and generalization vs abstraction:
  $\mathbb{R}^n$ vs vector space
Enriching perspectives from MathEd/PME (and CS)

Quinn’s No 2: Cognitive packaging: definitions plus exercises

- Deduction vs construction: building properties of abstract objects

- Concept definition vs concept image: focus on generic or otherwise disrupting images
Enriching perspectives from MathEd/PME (and CS)

Quinn’s No 3: Practice hard!

- Fluidity among images
  (eg, Dreyfus 1991; Tall 2013)

- Reification: point-wise vs. object-valued operators – focus enhancing (eg, Harel & Kaput 1991)
Some examples)

1. Concept definition vs concept image (generic images)
   - Fluidity among images
     (e.g., Dreyfus 1991; Tall 2013)
   - Reification: point-wise vs. object-valued operators – focus enhancing (e.g., Harel & Kaput 1991)
Some examples

1. Conceptual entities (reification)

- Fluidity among images
  (eg, Dreyfus 1991; Tall 2013)

- Reification: point-wise vs. object-valued operators – focus enhancing (eg, Harel & Kaput 1991)
Summary

1. While empirical findings lend support to Quinn’s claims about the adequacy of the mathematical practice as he describes it (as a three-step program), they also suggest that a more nuanced approach is advisable.

2. Empirical findings strongly suggest that MP includes a plethora of cognitive processes that go beyond deductive proof; therefore,

   PoM is dead, long live PoMP!
Thank You!